



Ribby with Wrea Endowed C.E Primary School

Calculations Policy

2022

Reviewed and Updated September 2022

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Although many of the KS2 objectives suggest that children should be using formal written methods, the National Curriculum document states:

"The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study."

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

There is no statutory requirement for particular methods of calculation to be used in schools. This policy highlights the methods chosen to be taught at Ribby with Wrea CE Primary School.

However, in the arithmetic test (introduced in May 2016), a method mark will be awarded if the final answer is incorrect and an acceptable method has been used, with only one mistake. Acceptable methods for long multiplication do not include grid method.

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Progression Towards a Written Method for Addition

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of subtraction
- commutative i.e. $5 + 3 = 3 + 5$
- associative i.e. $5 + 3 + 7 = 5 + (3 + 7)$

The fact that it is commutative and associative means that calculations can be rearranged, e.g. $4 + 13 = 17$ is the same as $13 + 4 = 17$.

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Mental recall of number bonds

$$6 + 4 = 10$$

$$\square + 3 = 10$$

$$25 + 75 = 100$$

$$19 + \square = 20$$

Use near doubles

$$6 + 7 = \text{double } 6 + 1 = 13$$

Addition using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

Early Learning Goal:

Using quantities and objects, children add two single-digit numbers and count on to find the answer.

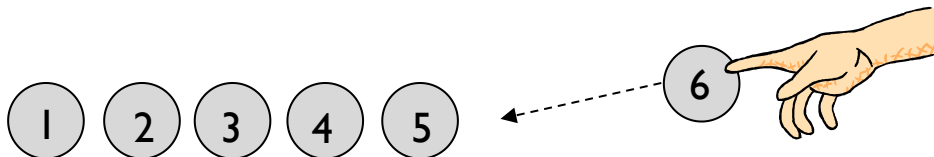
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Counting all method

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating $4 + 2$, they are encouraged to count out four counters and count out two counters.



To find how many altogether, touch and drag them into a line one at a time whilst counting.



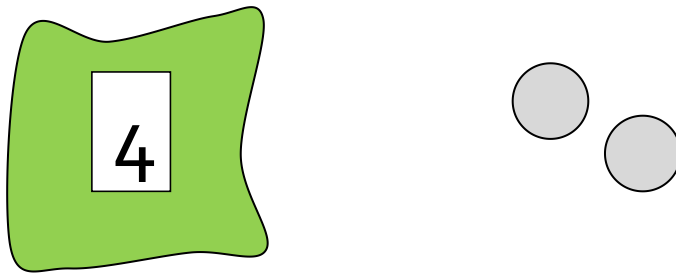
By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

Counting on method

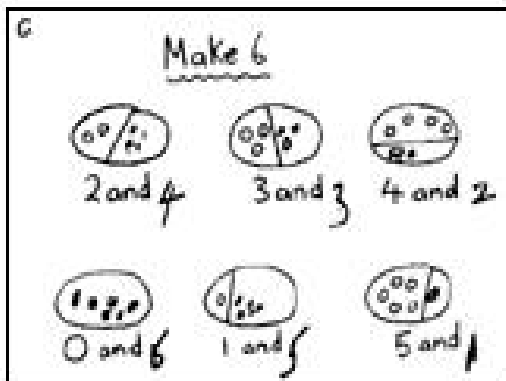
To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating $4 + 2$, count out the two groups of counters as before.



then put number in our head and count on.



Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



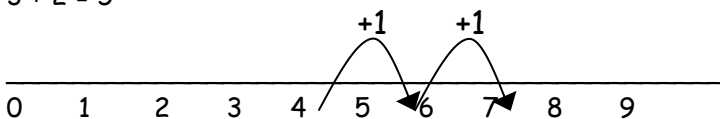
Those who are ready may record their own calculations.

Y1

End of Year Objective:
Add one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

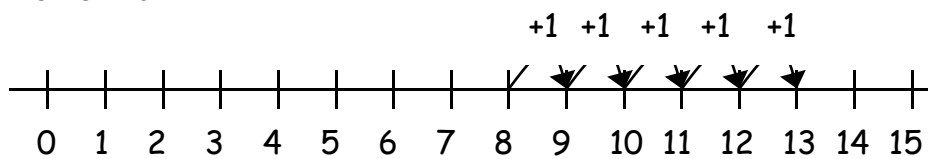
They use number lines and practical resources to support calculation and teachers demonstrate the use of the numberline.

$3 + 2 = 5$

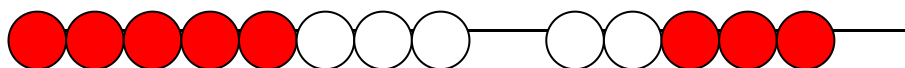


Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$$8 + 5 = 13$$



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

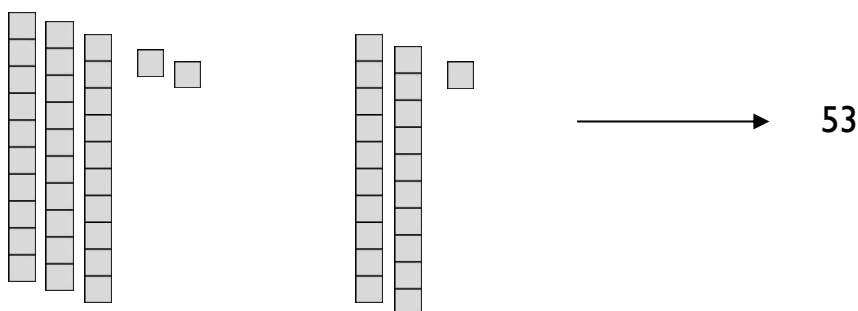


Y2

End of Year Objective:

Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit numbers.

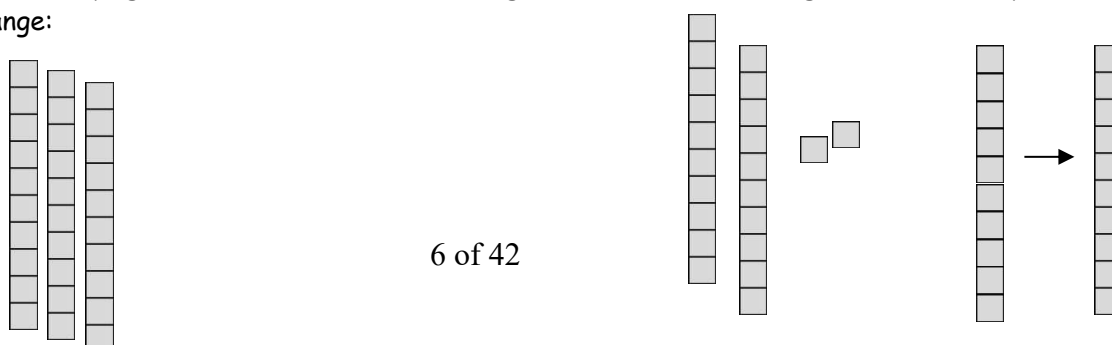
Children will use the Base 10 equipment or cubes to support their calculations. For example, to calculate $32 + 21$, they can make the individual amounts, counting the tens first and then count on the units.



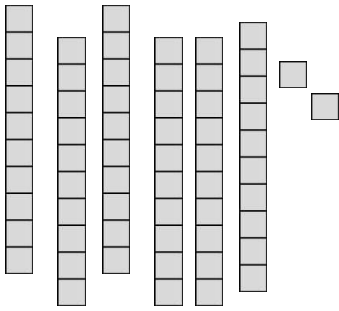
When the units total more than 10, children should be encouraged to exchange 10 units/ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition. For example, when calculating $35 + 27$, they can represent the amounts using Base 10 or cubes as shown:



Then, identifying the fact that there are enough units/ones to exchange for a ten, they can carry out this exchange:



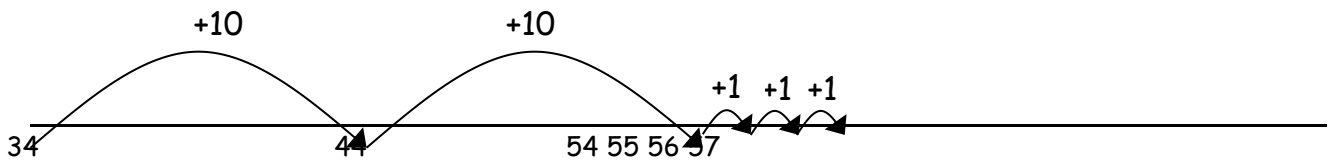
To leave:



Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

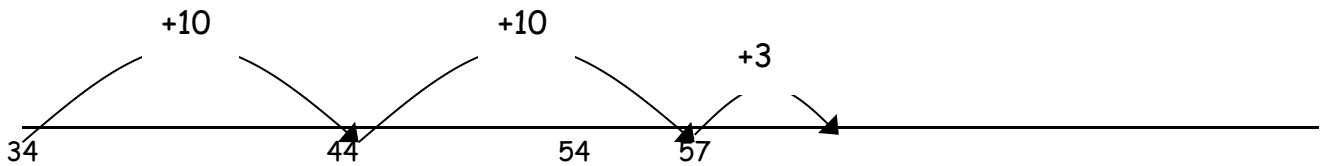
✓ First counting on in tens and ones.

$$34 + 23 = 57$$



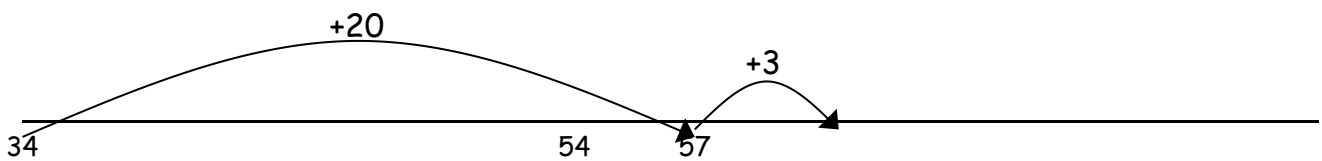
✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact $4 + 3 = 7$).

$$34 + 23 = 57$$



✓ Followed by adding the tens in one jump and the units in one jump.

$$34 + 23 = 57$$



Y3

End of Year Objective:
Add numbers with up to three digits, using formal written method of columnar addition.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange.

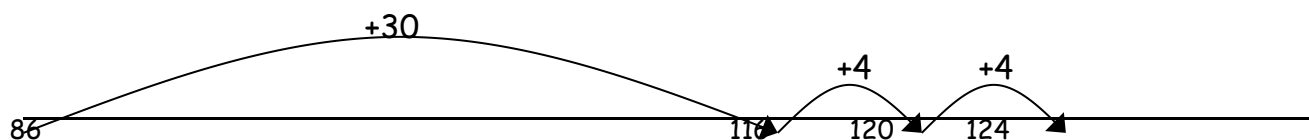
This should progress to children utilising the written and practical methods alongside each other and finally, and when they are ready, to children utilising just the written method.

By the end of year 3, children should also extend this method for three digit numbers.

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

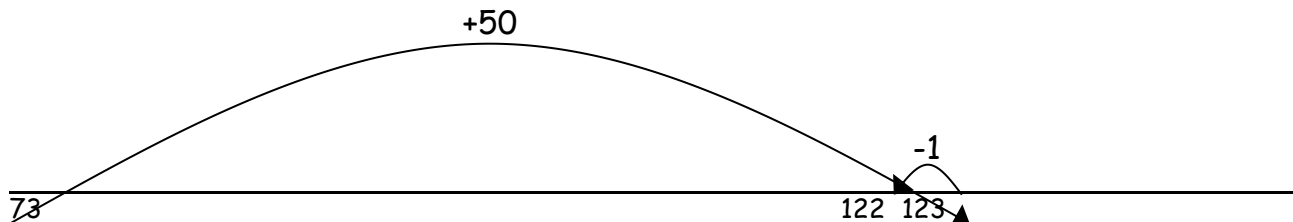
✓ Count on from the largest number irrespective of the order of the calculation.

$$38 + 86 = 124$$



✓ Compensation

$$49 + 73 = 122$$



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Children should add the **least significant digits** first (i.e. start with the units/ones), and in an identical method to that from year 2, should identify whether there are greater than ten units which can be exchanged for one ten.

Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside this practical method initially.

Written method

Step 1

$$\begin{array}{r} \text{T} \quad \text{U} \\ 6 \quad 5 \\ + \quad 2 \quad 7 \\ \hline \end{array}$$

Step 2

$$\begin{array}{r} \text{T} \quad \text{U} \\ 6 \quad 5 \\ + \quad 2 \quad 7 \\ \hline \quad \quad 2 \end{array}$$

Step 3

$$\begin{array}{r} \text{T} \quad \text{U} \\ 6 \quad 5 \\ + \quad 2 \quad 7 \\ \hline 9 \quad 2 \\ \hline \end{array}$$

Y4

End of Year Objective:

Add numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar addition where appropriate.

Children will move to year 4 using whichever method they were using as they transitioned from year 3.

Step 1

$$\begin{array}{r} \text{H T U} \\ 3 \ 6 \ 5 \\ + 2 \ 4 \ 7 \\ \hline \end{array}$$

Step 2

$$\begin{array}{r} \text{H T U} \\ 3 \ 6 \ 5 \\ + 2 \ 4 \ 7 \\ \hline \ 2 \\ \hline \text{I} \end{array}$$

Step 3

$$\begin{array}{r} \text{H T U} \\ 3 \ 6 \ 5 \\ + 2 \ 4 \ 7 \\ \hline \ 1 \ 2 \\ \hline \text{I I} \end{array}$$

Step 4

$$\begin{array}{r} \text{H T U} \\ 3 \ 6 \ 5 \\ + 2 \ 4 \ 7 \\ \hline 6 \ 1 \ 2 \\ \hline \text{I I} \end{array}$$

By the end of year 4, children should be using the written method confidently and with understanding. They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- decimals with one decimal place, knowing that the decimal points line up under one another.

Y5

End of Year Objective:

Add whole numbers with more than 4 digits *and decimals with two decimal places*, including formal written methods (columnar addition).

Children should continue to use the carrying method to solve calculations such as:

$$\begin{array}{r} 3 \ 3 \ 6 \ 4 \\ + \ 2 \ 4 \ 7 \\ \hline 3 \ 6 \ 1 \ 1 \\ \hline \text{I I} \end{array}$$

$$\begin{array}{r} 3 \ 1 \ 2 \ 1 \\ + \ 3 \ 7 \\ \hline 3 \ 3 \ 0 \ 6 \\ \hline \text{I I} \end{array}$$

$$\begin{array}{r} 3 \ . \ 5 \ 6 \\ + 2 \ . \ 4 \ 7 \\ \hline 6 \ . \ 0 \ 3 \\ \hline \text{I} \end{array}$$

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

Y6

End of Year Objective:
Add whole numbers and decimals using formal written methods (columnar addition).

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

$$\begin{array}{r}
 \\
 6 \\
 7 \\
 \\
 + \\
 \hline
 1 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 4 \\
 2 \\
 + 0 \\
 \hline
 4 2 \\
 \hline

 \end{array}$$

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

+ - + - + - + - + - + - +

Progression Towards a Written Method for Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of addition
- not commutative i.e. $5 - 3$ is not the same as $3 - 5$
- not associative i.e. $10 - 3 - 2$ is not the same as $10 - (3 - 2)$

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Mental recall of addition and subtraction facts

$$10 - 6 = 4$$

$$17 - \square = 11$$

$$20 - 17 = 3$$

$$10 - \square = 2$$

Find a small difference by counting up

$$82 - 79 = 3$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 - 52 = 34 \text{ (by counting back in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

YR

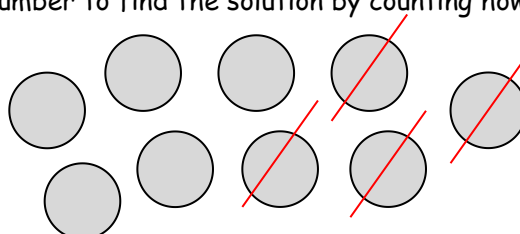
Early Learning Goal:

Using quantities and objects, children subtract two single-digit numbers and count on or back to find the answer.

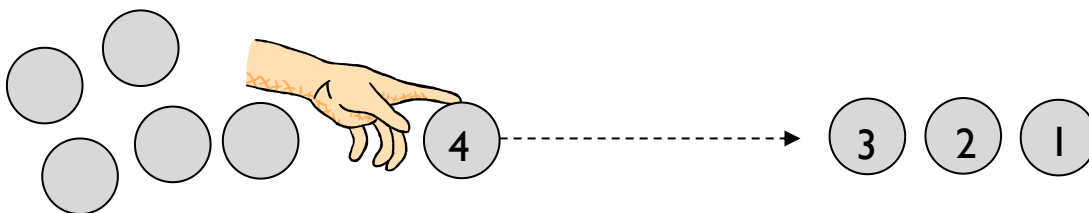
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Taking away

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g. $9 - 4$.



For illustration purposes, the amount being taken away are show crossed out. Children would be encouraged to physically remove these using touch counting.



By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

Those who are ready may record their own calculations.

Y1

End of Year Objective:
Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

Children will continue to use practical equipment and taking away strategies. To avoid the need to exchange for subtraction at this stage, it is advisable to continue to use equipment such as counters and cubes

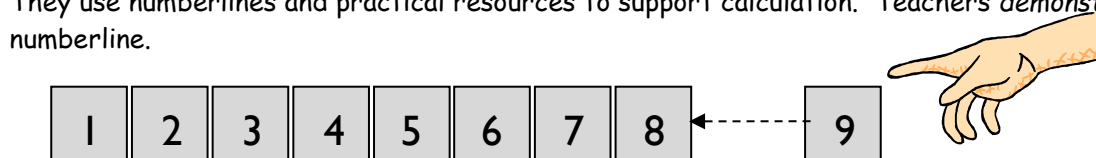


Touch count and remove the number to be taken away, in this case 4.

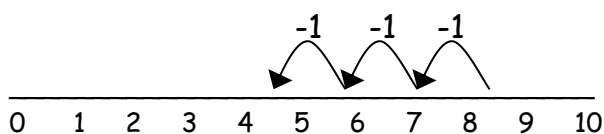


Touch count to find the number that remains.

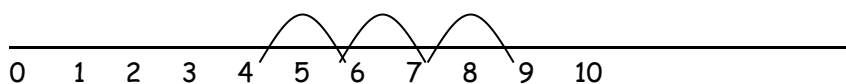
They use numberlines and practical resources to support calculation. Teachers demonstrate the use of the numberline.



$6 - 3 = 3$

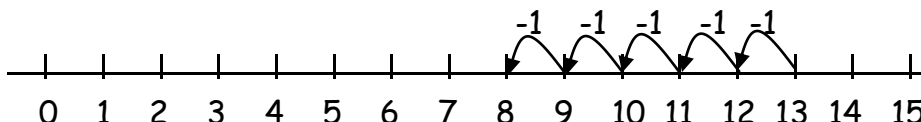


The numberline should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



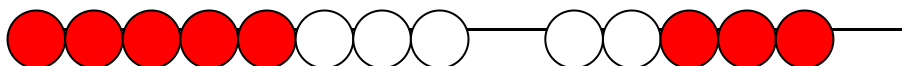
Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

$$13 - 5 = 8$$



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$

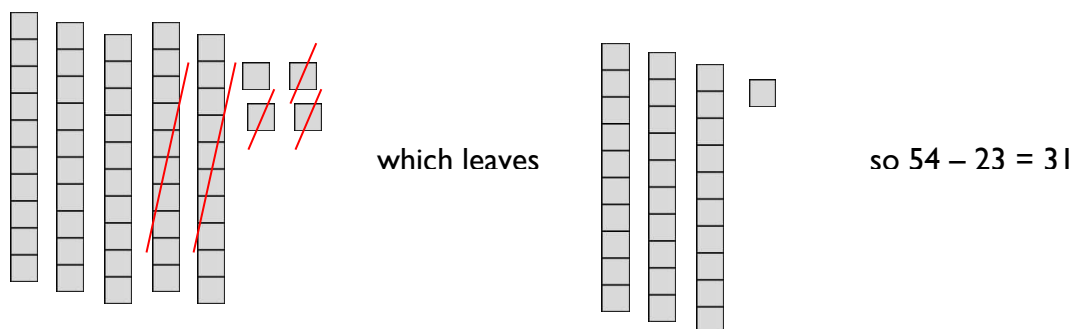


Y2

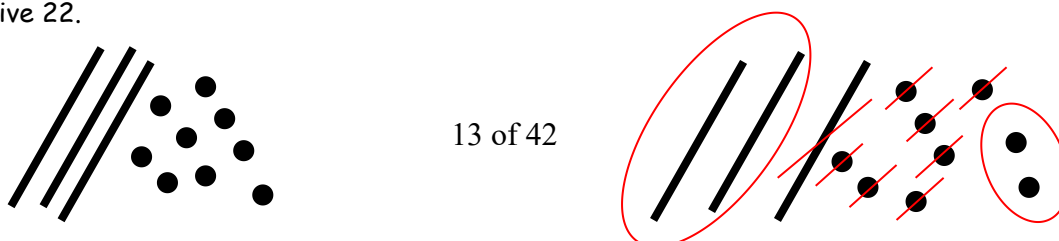
End of Year Objective:

Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers.

Children will begin to use the cubes and counters to support their calculations, still using a take away, or removal, method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate $54 - 23$, children would count out 54 using equipment (5 tens and 4 units). They need to consider whether there are enough units/ones to remove 3, in this case there are, so they would remove 3 units and then two tens, counting up the answer of 3 tens and 1 unit to give 31.



Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks), e.g. to calculate $39 - 17$ children would draw 39 as 3 tens (lines) and 4 units (dots) and would cross out 7 units and then one ten, counting up the answer of 2 tens and 2 units to give 22.



Circling the tens and units that remain will help children to identify how many remain.

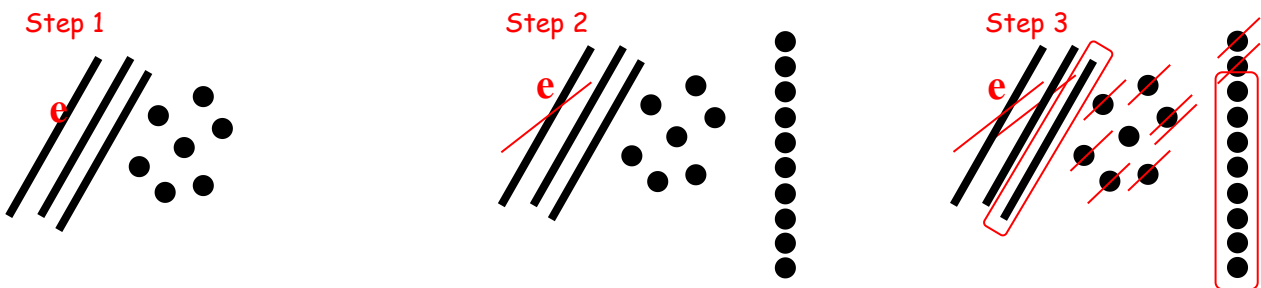
When the amount of units to be subtracted is greater than the units in the original number, an exchange method is required. This relies on children's understanding of ten units being an equivalent amount to one ten. To calculate $53 - 26$, by using practical equipment, they would count out 53 using the tens and units, as in Step 1. They need to consider whether there are enough units/ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.



The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.



When recording their own drawings, when calculating $37 - 19$, children would cross out a ten and exchange for ten units. The exchanged ten is denoted with an 'e' so children recognise this has not been subtracted. Drawing the units in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the units that were already in place.

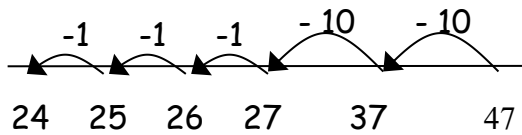


Children will begin to use empty number lines to support calculations.

Counting back

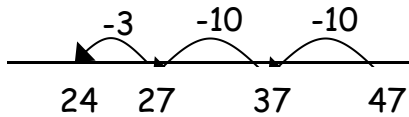
✓ First counting back in tens and ones.

$47 - 23 = 24$



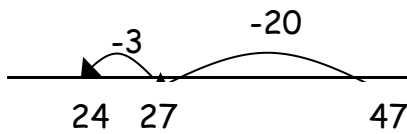
- ✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$).

$47 - 23 = 24$



- ✓ Subtracting the tens in one jump and the units in one jump.

$47 - 23 = 24$



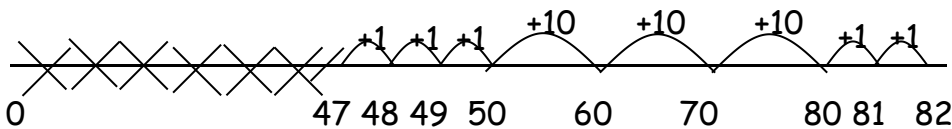
Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

$82 - 47$



Help children to become more efficient with counting on by:

- ✓ Subtracting the units in one jump;
- ✓ Subtracting the tens in one jump and the units in one jump;
- ✓ Bridging through ten.

End of Year Objective:
Subtract numbers with up to three digits, using formal written method of columnar subtraction.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children will build on their knowledge of using equipment from year 2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the units and tens as appropriate (as with the more informal method in year 2).

Step 1

$$\begin{array}{r} 80 \\ - 50 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ 7 \end{array}$$

Step 2

$$\begin{array}{r} 80 \\ - 50 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ 7 \end{array}$$

Step 3


$$\begin{array}{r} 80 \\ - 50 \\ \hline 30 \end{array} \quad \begin{array}{r} 9 \\ 7 \\ \hline 2 \end{array}$$

Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater.

This will be recorded by the children as:

$$\begin{array}{r} 80 \rightarrow 9 \\ - 50 \rightarrow 7 \\ \hline 30 \rightarrow 2 \end{array} = 32$$

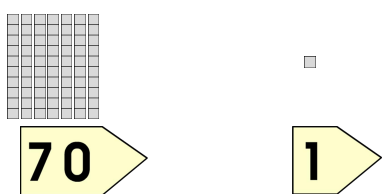
Children can also use jottings of the Base 10 materials (as in year 2) to support with their calculation, as in the example below.



$$\begin{array}{r}
 80 \rightarrow 9 \\
 - 50 \rightarrow 7 \\
 \hline
 30 \rightarrow 2 = 32
 \end{array}$$

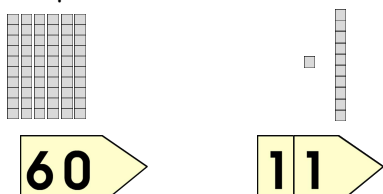
From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough units/ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same ($71 = 70 + 1 = 60 + 11$).

Step 1



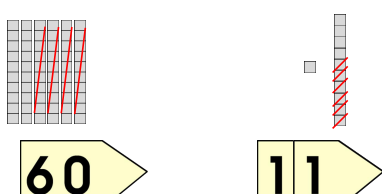
$$\begin{array}{r}
 70 \\
 - 40 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 1 \\
 - 6 \\
 \hline
 \end{array}$$

Step 2



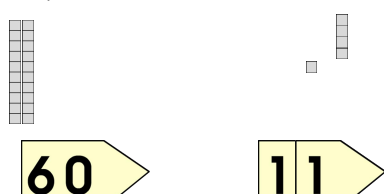
$$\begin{array}{r}
 60 \\
 - 40 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 11 \\
 - 6 \\
 \hline
 \end{array}$$

Step 3



$$\begin{array}{r}
 60 \\
 - 40 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 11 \\
 - 6 \\
 \hline
 \end{array}$$

Step 4



$$\begin{array}{r}
 60 \\
 - 40 \\
 \hline
 20
 \end{array}
 \quad
 \begin{array}{r}
 11 \\
 - 6 \\
 \hline
 5
 \end{array}$$

This will be recorded by the children as:

$$\begin{array}{r}
 60 \\
 70 \rightarrow 1 \\
 - 40 \rightarrow 6 \\
 \hline
 20 \rightarrow 5 = 25
 \end{array}$$

By the end of year 3, children should also extend this method for three digit numbers.

Children will continue to use empty number lines with increasingly large numbers.

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Partitioning and decomposition

This process should be demonstrated using arrow cards to show the partitioning.

NOTE When solving the calculation $89 - 57$, children should know that 57 **does NOT EXIST AS AN AMOUNT** it is what you are subtracting from the other number.

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 + 9 \\ 50 + 7 \\ \hline 30 + 2 = 32 \end{array}$$

Initially, the children will be taught using examples that do not need the children to exchange.

From this the children will begin to exchange.

$$\begin{array}{r} 71 \\ - 46 \\ \hline \end{array} = \quad =$$

Step 1
$$\begin{array}{r} 70 + 1 \\ - 40 + 6 \\ \hline \end{array}$$

Step 2
$$\begin{array}{r} 60 + 11 \\ - 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$

The calculation should be read as e.g. take 6 from 1.

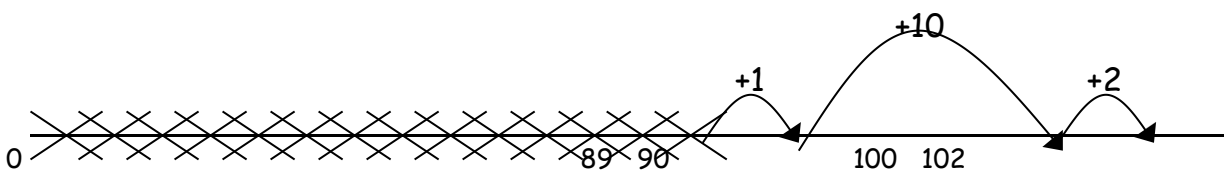
This would be recorded by the children as

$$\begin{array}{r} 60 \\ \cancel{70} + \cancel{1} \\ - \cancel{40} + \cancel{6} \\ \hline 20 + 5 = 25 \end{array}$$

Children should know that units line up under units, tens under tens, and so on.

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$102 - 89 = 13$



Y4

End of Year Objective:

Subtract numbers with up to 4 digits *and* decimals with one decimal place using the formal written method of columnar subtraction where appropriate.

Children will move to year 4 using whichever method they were using as they transitioned from year 3.

Step 1

$$\begin{array}{r} 700 \rightarrow 50 \rightarrow 4 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

Step 2 (exchanging from tens to units)

$$\begin{array}{r} 700 \rightarrow \overset{40}{\cancel{50}} \rightarrow 14 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

Step 3 (exchanging from hundreds to tens)

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow 14 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

Step 4

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow 14 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

This would be recorded by the children as:

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow 14 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

When children are ready, this leads on to the compact method of decomposition:

$$\begin{array}{r} \overset{6}{4} \overset{14}{\cancel{5}} 14 \\ - 3286 \\ \hline 1468 \end{array}$$

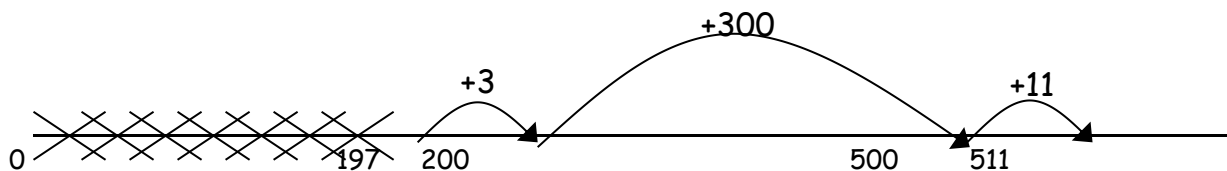
By the end of year 4, children should be using the written method confidently and with understanding. They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- decimals with one decimal place, knowing that the decimal points line up under one another.

NB If the children have reached the concise stage they will then continue this method through into years 5 and 6. They will not go back to using the expanded methods.

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$511 - 197 = 314$$



Y5

End of Year Objective:
Subtract whole numbers with more than 4 digits and decimals with two decimal places, including formal written methods (columnar subtraction).

Children should continue to use the decomposition method to solve calculations such as:

$$\begin{array}{r} \overset{6}{\cancel{7}} \overset{6}{\cancel{10}} \\ - \quad 3 \quad 2 \quad 2 \quad 6 \\ \hline 3 \quad 8 \quad 4 \quad 6 \end{array}$$

$$\begin{array}{r} \overset{2}{\cancel{3}} \overset{13}{\cancel{4}} \overset{12}{\cancel{12}} \\ - \quad 1 \quad . \quad 7 \quad 6 \\ \hline 1 \quad . \quad 6 \quad 6 \end{array}$$

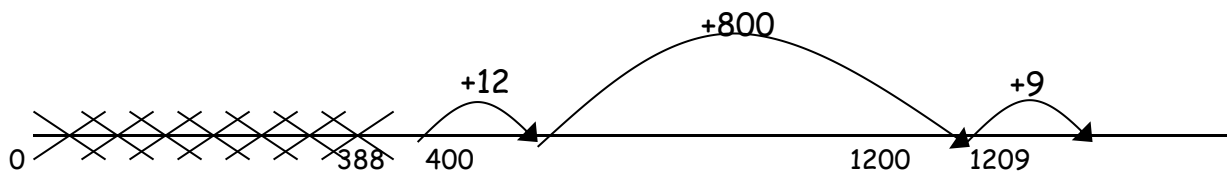
They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

NB *If the children have reached the concise stage they will then continue this method through into year 6. They will not go back to using the expanded methods.*

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$1209 - 388 = 821$$



End of Year Objective:
Subtract whole numbers and decimals using formal written methods (columnar subtraction).

Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.

$$\begin{array}{r}
 \begin{array}{cccc}
 & 5 & 13 & \\
 \cancel{6} & \cancel{4} & 13 & 2 \\
 - & 4 & 6 & 8 & 1 \\
 \hline
 & 1 & 7 & 5 & 1
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{cccc}
 & 3 & 6 & 11 & \\
 \cancel{4} & 11 & \cancel{7} & . & \cancel{2} & 10 \\
 - & 3 & 4 & . & 7 & 1 \\
 \hline
 & 3 & 8 & 2 & . & 4 & 9
 \end{array}
 \end{array}$$

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

+ - + - + - + - + - + - +

Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of division
- commutative i.e. 5×3 is the same as 3×5
- associative i.e. $2 \times 3 \times 5$ is the same as $2 \times (3 \times 5)$

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Doubling and halving

Applying the knowledge of doubles and halves to known facts.

e.g. 8×4 is double 4×4

Using multiplication facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table
 5 times table
 10 times table

Year 3 2 times table
 3 times table
 4 times table
 5 times table
 6 times table
 10 times table

Year 4 Derive and recall all multiplication facts up to 12×12

Years 5 & 6 Derive and recall quickly all multiplication facts up to 12×12 .

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\,000$, $0.3 \times 7 = 2.1$ etc

Use closely related facts already known

$13 \times 11 = (13 \times 10) + (13 \times 1)$
 = $130 + 13$
 = 143

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.
Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

Partitioning

$$\begin{aligned} 23 \times 4 &= (20 \times 4) + (3 \times 4) \\ &= 80 + 12 \\ &= 102 \end{aligned}$$

Use of factors

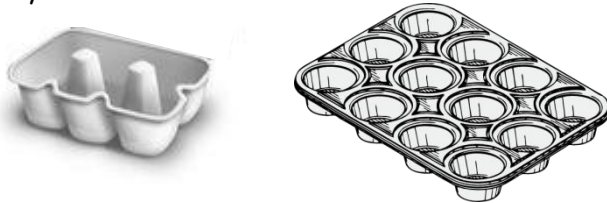
$$8 \times 12 = 8 \times 4 \times 3$$

YR

Early Learning Goal:
Children solve problems, including doubling.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

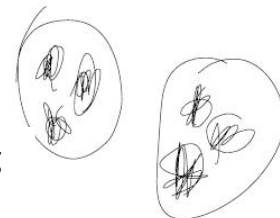
Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing the fingers on each hand as a double.



A child's jotting showing double three as three cookies on each plate.

Y1

End of Year Objective:
Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

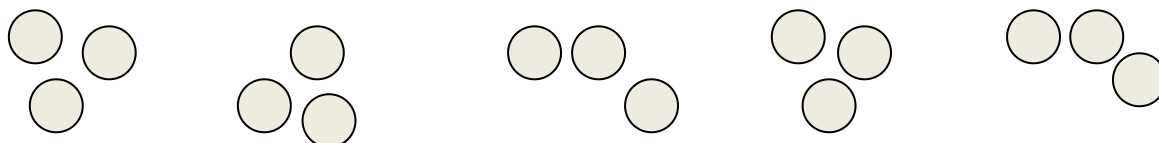
Children will experience equal groups of objects and will count in 2s, 5s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.

Y2

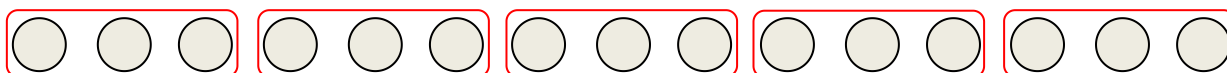
End of Year Objective:
Calculate mathematical statements for multiplication (using repeated addition) and write them using the multiplication (x) and equals (=) signs.

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

5×3 can be shown as five groups of three with counters, either grouped in a random pattern, as below:

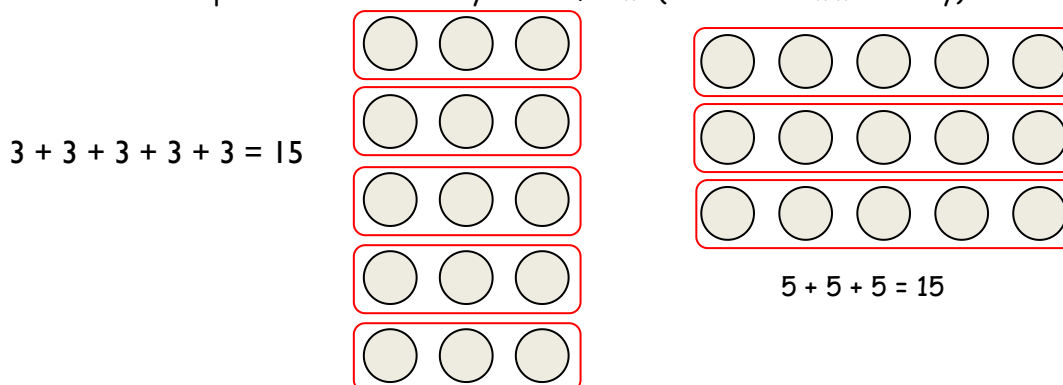


or in a more ordered pattern, with the groups of three indicated by the border outline:



Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

$5 \times 3^*$ can be represented as an array in two forms (as it has commutativity):



*For mathematical accuracy 5×3 is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. Once children understand the commutative order of multiplication the order is irrelevant).

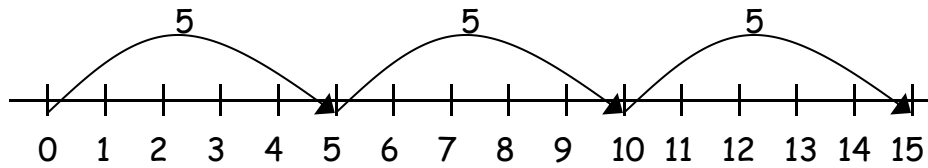
Children will develop their understanding of multiplication and use jottings to support calculation:

✓ **Repeated addition**

3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

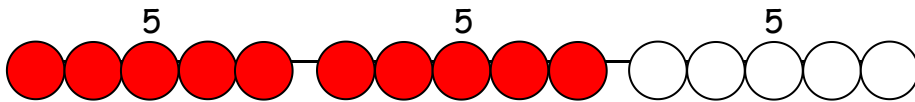
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



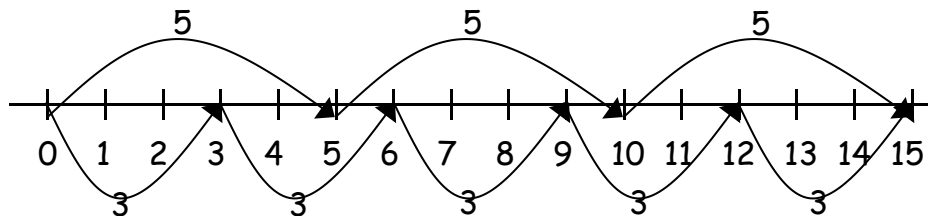
and on a bead bar:

$$5 \times 3 = 5 + 5 + 5$$



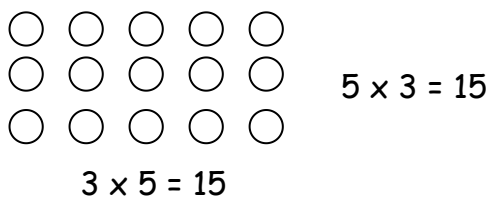
✓ Commutativity

Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



✓ Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



Y3

End of Year Objective:

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.*

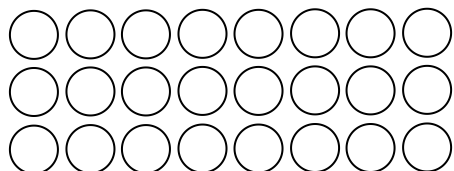
**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$3 \times 8$$

They may show this using practical equipment:



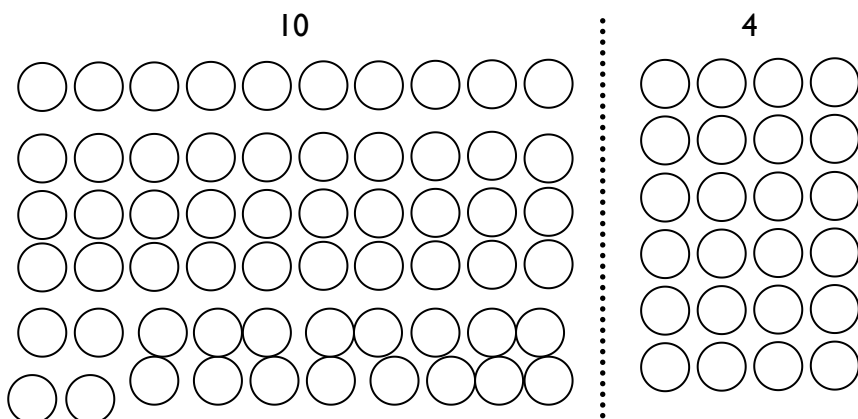
$$3 \times 8 = 8 + 8 + 8 = 24$$

or by jottings using squared paper:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|--|
| | x | x | x | x | x | x | x | x | |
| x | x | x | x | x | x | x | x | x | |
| x | x | x | x | x | x | x | x | x | |
| | | | | | | | | | |

$$3 \times 8 = 8 + 8 + 8 = 24$$

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating 14×6 , children should set out the array, then partition the array so that one array has ten columns and the other four.

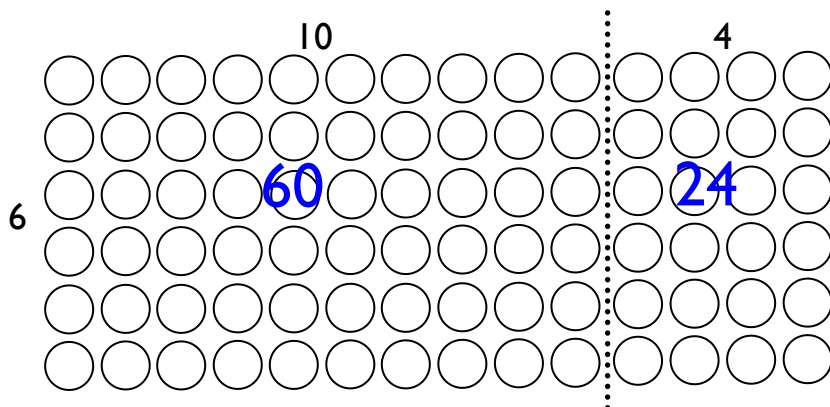


Partitioning in this way, allows children to identify that the first array shows 10×6 and the second array shows 4×6 . These can then be added to calculate the answer:

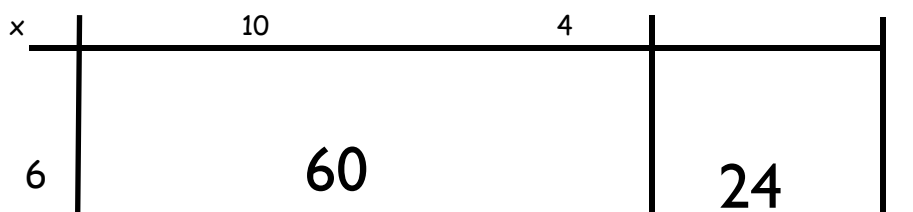
$$\begin{aligned} &(6 \times 10) + (6 \times 4) \\ = &60 + 24 \\ = &84 \end{aligned}$$

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

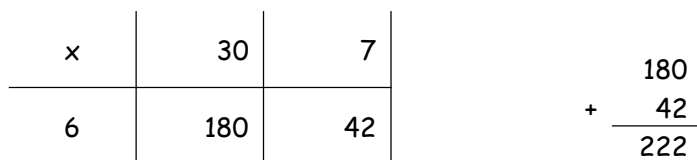
From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

13 x 8



When children are ready, they can then progress to using this method with other two-digit numbers.

37 x 6

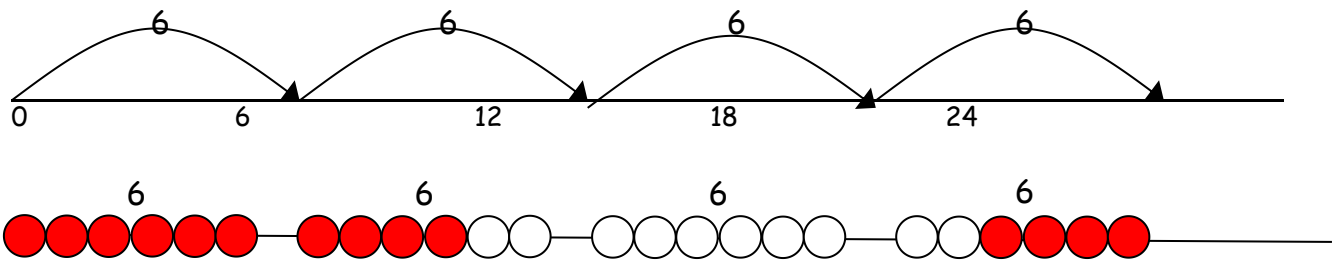


Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Repeated addition

4 times 6 is $6 + 6 + 6 + 6 = 24$ or 4 lots of 6 or 6×4

Children should use number lines or bead bars to support their understanding.



Children will also develop an understanding of

✓ Scaling

e.g. Find a ribbon that is 4 times as long as the blue ribbon



✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times \circ = 32$$

✓ Partitioning

$$\begin{aligned} 38 \times 5 &= (30 \times 5) + (8 \times 5) \\ &= 150 + 40 \\ &= 190 \end{aligned}$$

Y4

End of Year Objective:
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Children will move to year 4 using whichever method they were using as they transitioned from year 3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

$$79 \times 8$$

| | | |
|---|-----|----|
| x | 70 | 9 |
| 8 | 560 | 72 |

$$\begin{array}{r} 560 \\ + 72 \\ \hline 632 \end{array}$$

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that $7 \times 8 = 56$ to know that $70 \times 8 = 560$.

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

$$346 \times 8$$

| | | | |
|---|------|-----|----|
| x | 300 | 40 | 6 |
| 8 | 2400 | 320 | 48 |

$$\begin{array}{r} 2400 \\ + 320 \\ + 48 \\ \hline 2768 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y5

End of Year Objective:

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

$$4346 \times 8$$

| | | | | |
|---|--------|------|-----|----|
| x | 4 000 | 300 | 40 | 6 |
| 8 | 32 000 | 2400 | 320 | 48 |

$$\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$$

and numbers with up to four digits by a two-digit number, e.g.

$$2693 \times 24$$

| | | | | |
|----|-------|-------|------|----|
| x | 2000 | 600 | 90 | 3 |
| 20 | 40000 | 12000 | 1800 | 60 |
| 4 | 8000 | 2400 | 360 | 12 |

$$\begin{array}{r} 40000 \\ + 8000 \\ + 12000 \\ + 2400 \\ + 1800 \\ + 360 \\ + 60 \\ + 12 \\ \hline 64632 \end{array}$$

The long list of numbers in the addition part can be used to check that all of the answers from the grid have been included, however, when children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they should be encouraged to do so. For example,

| | | | | | |
|----|-------|-------|------|----|---------------|
| x | 2000 | 600 | 90 | 3 | |
| 20 | 40000 | 12000 | 1800 | 60 | = 53 860 |
| 4 | 8000 | 2400 | 360 | 12 | = 10 772 + |
| | | | | | <u>64 632</u> |

Adding across mentally, leads children to finding the separate answers to:

2 693 x 20
2 693 x 4

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

During Year 5, the transition from the grid method into the formal vertical method for multiplication should take place. The traditional vertical compact method of written multiplication is a highly efficient way to calculate, but it has a very condensed form and needs to be introduced carefully.

It is most effective to begin with the grid method, moving to an expanded vertical layout, before introducing the compact form. This allows children to see, and understand, how the processes relate to each other and where the individual multiplication answers come from e.g.

368 x 6

| | | | |
|---|-------|-----|----|
| x | 300 | 60 | 8 |
| 6 | 1 800 | 360 | 48 |

$$\begin{array}{r} 1800 \\ + 360 \\ + 48 \\ \hline 2208 \end{array}$$

| |
|---------------------|
| Th H T U |
| 3 6 8 |
| x 6 |
| 4 8 (8 x 6) |
| 3 6 0 (60 x 6) |
| + 1 8 0 0 (300 x 6) |
| <u>2 2 0 8</u> |

| |
|---------------------|
| Th H T U |
| 3 6 8 |
| x 6 |
| 4 8 (8 x 6) |
| 3 6 0 (60 x 6) |
| + 1 8 0 0 (300 x 6) |
| <u>2 2 0 8</u> |

becomes

| |
|----------------|
| Th H T U |
| 3 6 8 |
| x 6 |
| <u>2 2 0 8</u> |
| 4 4 |

The place value columns are labelled to ensure children understand the size of the partitioned digits in the original number(s) and in the answer.

It is vital that the teacher models the correct language when explaining the process of the compact method. The example shown should be explained as:

"Starting with the least significant digit... 8 multiplied by 6 is 48, put 8 in the units and carry 4 tens (40). 6 tens multiplied by 6 are 36 tens. Add the 4 tens carried over to give 40 tens (which is the same as 4 hundreds and 0 tens). Put 0 in the tens place of the answer and carry 4 hundreds.

3 hundreds multiplied by 6 are 18 hundreds. Add the 4 hundreds carried over to give 22 hundreds (which is the same as 2 thousands and 2 hundreds). Write 2 in the hundreds place of the answer and 2 in the thousands place of the answer."

Children should recognise that the answer is close to an estimated answer of $400 \times 6 = 2\,400$

Long multiplication could also be introduced by comparing the grid method with the compact vertical method. Mentally totalling each row of answers is an important step in children making the link between the grid method and the compact method.

| | | | | |
|----|-------|------|----|---------------|
| x | 600 | 90 | 3 | |
| 20 | 12000 | 1800 | 60 | = 13 860 |
| 4 | 2400 | 360 | 12 | = 2 772 + |
| | | | | <u>16 632</u> |

Children should only be expected to move towards this next method if they have a secure understanding of place value. It is difficult to explain the compact method without a deep understanding of place value.

The example shown should be explained as:

Step 1

$$\begin{array}{r}
 \text{TTh Th H T U} \\
 6\ 9\ 3 \\
 \times 2\ 4 \\
 \hline
 2\ 7\ 7\ 2 \\
 3\ \\
 \hline
 \end{array}
 \quad (693 \times 4)$$

"Starting with the least significant digit... 3 multiplied by 4 is 12; put 2 in the units and carry 1 ten (10).

9 tens multiplied by 4 are 36 tens. Add the 1 ten carried over to give 37 tens (which is the same as 3 hundreds and 7 tens). Put 7 in the tens place of the answer and carry 3 hundreds.

6 hundreds multiplied by 4 are 24 hundreds. Add the 3 hundreds carried over to give 27 hundreds (which is the same as 2 thousands and 7 hundreds). Write 7 in the hundreds place of the answer and 2 in the thousands place of the answer. We have now found the answer to 693×4 . Step 1 is complete so to avoid confusion later, we will cross out the carried digits 3 and 1."

Notice this answer can clearly be seen in the grid method example.

Step 2

$$\begin{array}{r}
 \text{TTh Th H T U} \\
 6\ 9\ 3 \\
 \times 2\ 4 \\
 \hline
 2\ 7\ 7\ 2 \\
 13\ 8\ 6\ 0 \\
 \hline
 \end{array}
 \quad \begin{array}{l}
 (693 \times 4) \\
 (693 \times 20)
 \end{array}$$

Now we are multiplying 693 by 20. Starting with the least significant digit of the top number... 3 multiplied by 20 is 60. Write this answer in.

90 multiplied by 20 is 1 800. There are no units and no tens in this answer, so write 8 in the hundreds place and carry 1 in the thousands.

600 multiplied by 20 is 12 000. Add the 1 (thousand) that was carried to give 13 000. There are no units, no tens and no hundreds in this answer, so write 3 in the thousands place and 1 in the ten thousands place.

Step 3

$$\begin{array}{r}
 \text{TTh Th H T U} \\
 6\ 9\ 3 \\
 \times 2\ 4 \\
 \hline
 2\ 7\ 7\ 2 \\
 13\ 8\ 6\ 0 \\
 \hline
 1\ 6\ 6\ 3\ 2 \\
 \hline
 \end{array}
 \quad \begin{array}{l}
 (693 \times 4) \\
 (693 \times 20)
 \end{array}$$

The final step is to total both answers using efficient columnar addition.

When using the compact method for long multiplication, all carried digits should be placed below the line of that answer e.g. 3×4 is 12, so the 2 is written in the units column and the 10 is carried as a small 1 in the tens column.

This carrying below the answer is in line with the written addition policy in which carried digits are always written below the answer/line.

Y6

End of Year Objective:

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

Multiply one-digit numbers with up to two decimal places by whole numbers.

By the end of year 6, children should be able to use the grid method and the compact method to multiply any number by a two-digit number. They could also develop the method to be able to multiply decimal numbers with up to two decimal places, but having been introduced to expanded and compact vertical methods in Year 5, it may be appropriate to use the expanded vertical method when introducing multiplication involving decimals.

$$4.92 \times 3$$

| | | |
|--|---------|--|
| $\begin{array}{r} \text{T U . t h} \\ 4 . 9 2 \\ \times \quad 3 \\ \hline 0 . 0 6 \quad (0.02 \times 3) \\ 2 . 7 \quad (0.9 \times 3) \\ + 1 2 \quad (4 \times 3) \\ \hline 1 4 . 7 6 \end{array}$ | becomes | $\begin{array}{r} \text{T U . t h} \\ 4 . 9 2 \\ \times \quad 3 \\ \hline 1 4 . 7 6 \end{array}$ |
|--|---------|--|

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

ThHTU x U

(Short multiplication - multiplication by a single digit)

$$4346 \times 8$$

Children will approximate first

$$4346 \times 8 \text{ is approximately } 4346 \times 10 = 43460$$

| | | | | | |
|---|-------|------|-----|----|---|
| x | 4000 | 300 | 40 | 6 | |
| 8 | 32000 | 2400 | 320 | 48 | $\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$ |

+ - + - + - + - + - + - +

Progression Towards a Written Method for Division

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of multiplication
- not commutative i.e. $15 \div 3$ is not the same as $3 \div 15$
- not associative i.e. $30 \div (5 \div 2)$ is not the same as $(30 \div 5) \div 2$

MENTAL CALCULATIONS

(ongoing)

These are a **selection** of mental calculation strategies:

Doubling and halving

Knowing that halving is dividing by 2

Deriving and recalling division facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table

5 times table

10 times table

Year 3 2 times table

3 times table

4 times table

5 times table

6 times table

10 times table

Year 4 Derive and recall division facts for all tables up to 12×12

Year 5 & 6 Derive and recall quickly division facts for all tables up to 12×12

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use of factors

$$378 \div 21$$

$$378 \div 3 = 126$$

$$378 \div 21 = 18$$

$$126 \div 7 = 18$$

Use related facts

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$, or $1.54 \div 1.1$?

YR

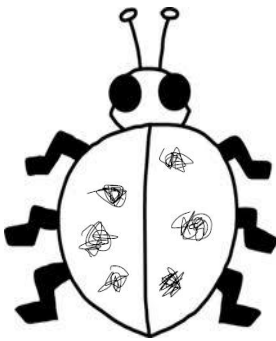
Early Learning Goal:
Children solve problems, including halving and sharing.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

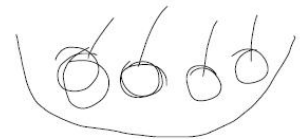
Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



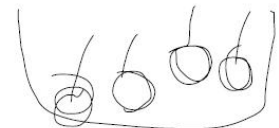
They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.



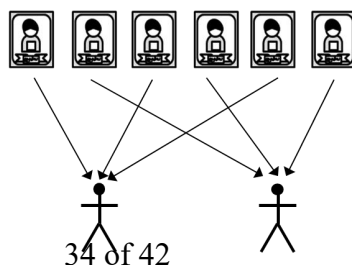
A child's jotting showing how they shared the apples at snack time between two groups.



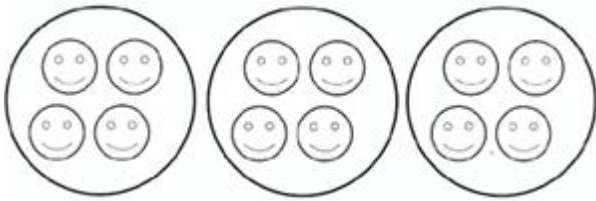
Y1

End of Year Objective:
Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.



Y2

End of Year Objective:
Calculate mathematical statements for division within the multiplication tables and write them using the division (\div) and equals (=) signs.

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

$12 \div 3 =$

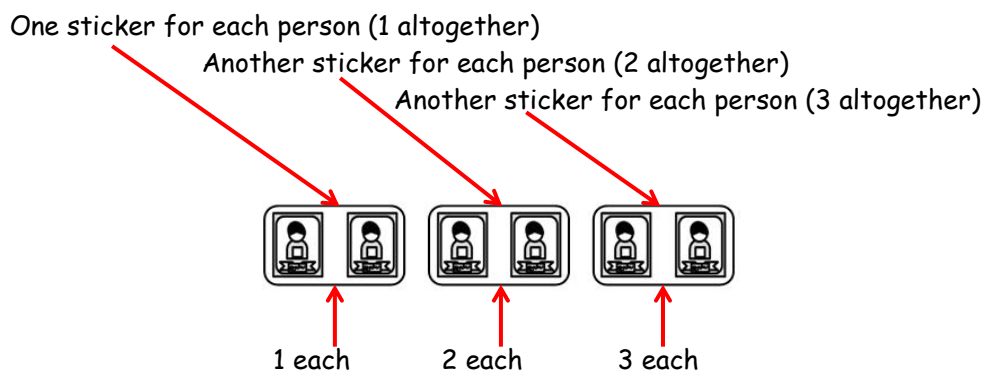


Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

The link between sharing and grouping can be modelled in the following way:

To solve the problem 'If six football stickers are shared between two people, how many do they each get?'

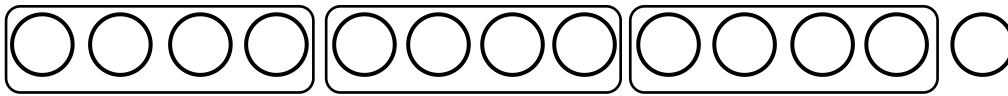
Place the football stickers in a bag or box and ask the children how many stickers would need to be taken out of the box to give each person one sticker each (i.e. 2) and exemplify this by putting the cards in groups of 2 until all cards have been removed from the bag.



Or:

Children should also continue to develop their knowledge of division with remainders, e.g.

$$13 \div 4 =$$



$$13 \div 4 = 3 \text{ remainder } 1$$

Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation $13 \div 4$, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

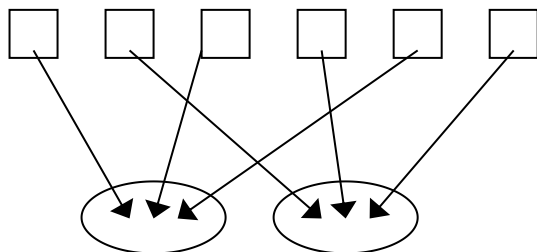
I have £13. Books are £4 each. How many can I buy?
 Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?
 Answer: 4 (the remaining 1 apple still need to be placed into a box)

Children will develop their understanding of division and use jottings to support calculation

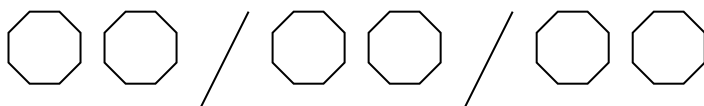
✓ Sharing equally

6 sweets shared between 2 people, how many do they each get?



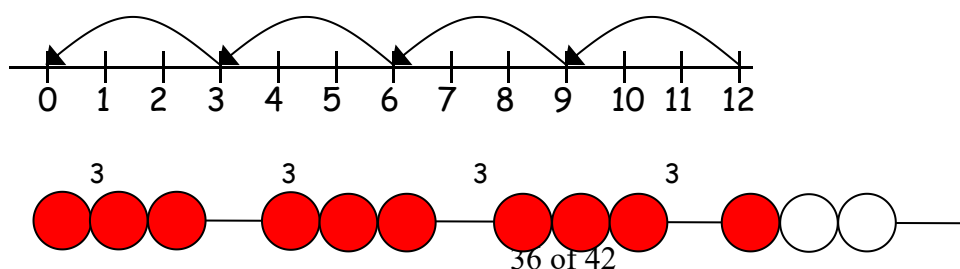
✓ Grouping or repeated subtraction

There are 6 sweets, how many people can have 2 sweets each?



✓ Repeated subtraction using a number line or bead bar

$$12 \div 3 = 4$$



The bead bar will help children with interpreting division calculations such as $10 \div 5$ as 'how many 5s make 10?'

✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$\square \div 2 = 4$ $20 \div \triangle = 4$ $\square \div \triangle = 4$

Y3

End of Year Objective:
Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study."*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$43 \div 8 =$



$43 \div 8 = 5 \text{ remainder } 3$

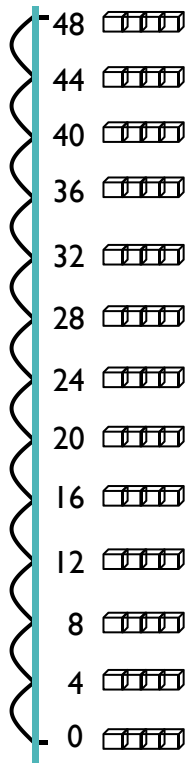
In preparation for developing the 'chunking' method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

Stage 1 - repeatedly subtracting individual groups of the divisor

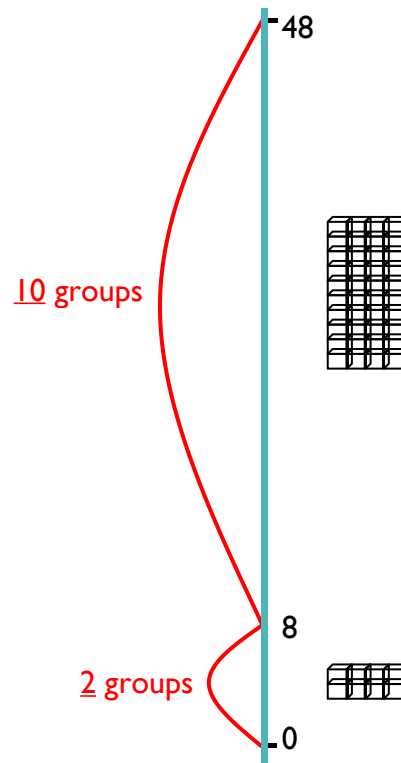
Stage 2 - subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

Stage 1
 $48 \div 4 = 12$ (groups of 4)



Stage 2
 $48 \div 4 = 10$ (groups of 4) + 2 (groups of 4)
 $= 12$ (groups of 4)



Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

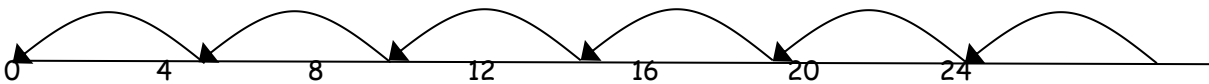
Ensure that the emphasis in Y3 is on grouping rather than sharing.

Children will continue to use:

✓ **Repeated subtraction using a number line**

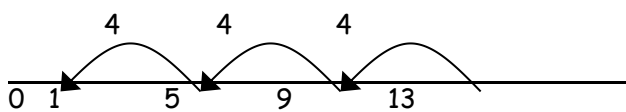
Children will use an empty number line to support their calculation.

$24 \div 4 = 6$



Children should also move onto calculations involving remainders.

$13 \div 4 = 3 \text{ r } 1$



✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

$26 \div 2 = \square$

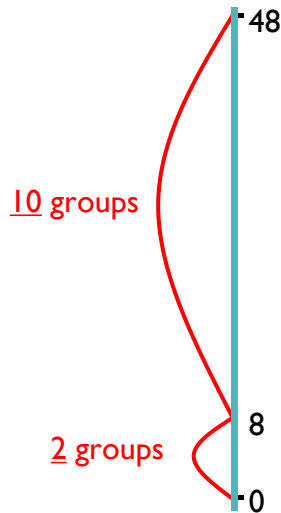
$24 \div \triangle = 12$

$\square \div 10 = 8$

End of Year Objective:

Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.



$$\begin{array}{r}
 12 \\
 4 \overline{) 48} \\
 \underline{- 40} \quad 10x \\
 8 \\
 \underline{- 8} \quad 2x \\
 0 \\
 \text{Answer: } 12
 \end{array}$$

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding.

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. Any remainders should be shown as integers, e.g.

73 ÷ 3

$$\begin{array}{r}
 24r1 \\
 3 \overline{) 73} \\
 \underline{- 30} \quad 10x \\
 43 \\
 \underline{- 30} \quad 10x \\
 13 \\
 \underline{- 6} \quad 2x \\
 7 \\
 \underline{- 6} \quad 2x \\
 1
 \end{array}$$

Key facts box

| | |
|-----|----|
| 1x | 3 |
| 2x | 6 |
| 5x | 15 |
| 10x | 30 |

By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

196 ÷ 6

$$\begin{array}{r}
 32r4 \\
 3 \overline{) 196} \\
 \underline{- 120} \quad 20x \\
 76 \\
 \underline{- 60} \quad 10x \\
 16 \\
 \underline{- 12} \quad 2x \\
 4
 \end{array}$$

Key facts box

| | |
|-----|-----|
| 1x | 6 |
| 2x | 12 |
| 4x | 24 |
| 5x | 30 |
| 10x | 60 |
| 20x | 120 |

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example $62 \div 8$ is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?
 Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?
 Answer: 8 (the remaining 6 apples still need to be placed into a box)

Y5

End of Year Objective:
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. During Year 5, children should be encouraged to be efficient when using the chunking method and not have any subtraction steps that repeat a previous step. For example, when performing $347 \div 8$ an initial subtraction of 160 (20×8) and a further subtraction of 160 (20×8) should be changed to a single subtraction of 320 (40×8). Also, any remainders should be shown as integers, e.g.

$523 \div 8$

$$\begin{array}{r}
 \overline{) 523} \\
 \underline{- 320} \quad 40x \\
 203 \\
 \underline{- 160} \quad 20x \\
 43 \\
 \underline{- 40} \quad 5x \\
 \hline
 \overline{) 523} = 65 \text{ r}3
 \end{array}$$

By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x.

$2458 \div 7$

$$\begin{array}{r}
 \overline{) 2458} \\
 \underline{- 2100} \quad 300x \\
 358 \\
 \underline{- 350} \quad 50x \\
 8 \\
 \underline{- 7} \quad 1x \\
 \hline
 \overline{) 2458} = 351 \text{ r}1
 \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Y6

End of Year Objective:
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.
Use written division methods in cases where the answer has up to two decimal places.

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

6367 ÷ 28

$$\begin{array}{r}
 227r11 \\
 28 \overline{)6367} \\
 \underline{-5600} \quad 200x \\
 767 \\
 \underline{-560} \quad 20x \\
 207 \\
 \underline{-140} \quad 5x \\
 67 \\
 \underline{-56} \quad 2x \\
 11
 \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places.

This should first be demonstrated using a simple calculation such as 13 ÷ 4 to show the remainder initially as a fraction.



Using practical equipment, children can see that for 13 ÷ 4, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is $\frac{1}{4}$. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

3574 ÷ 8

$$\begin{array}{r}
 8 \overline{)3574} \\
 \underline{-3200} \quad 400x \\
 374 \\
 \underline{-320} \quad 40x \\
 54 \\
 \underline{-48} \quad 6x \\
 6
 \end{array}$$

$$\frac{6}{8} \leftarrow \begin{array}{l} \text{remainder} \\ \text{divisor} \end{array}$$

So 3574 ÷ 8 is $446\frac{6}{8}$
 41 of 41 (when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2} = 0.5$

Quarters: $\frac{1}{4} = 0.25, \frac{3}{4} = 0.75$

Fifths: $\frac{1}{5} = 0.2, \frac{2}{5} = 0.4, \frac{3}{5} = 0.6, \frac{4}{5} = 0.8$

Tenths: $\frac{1}{10} = 0.1, \frac{2}{10} = 0.2, \frac{3}{10} = 0.3, \frac{4}{10} = 0.4, \frac{5}{10} = 0.5, \frac{6}{10} = 0.6, \frac{7}{10} = 0.7, \frac{8}{10} = 0.8, \frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

In the example above, $3574 \div 8$, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms. As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 446.75.

$87.5 \div 7$

| | |
|----------|------|
| 12.5 | |
| 7) 87.5 | |
| - 70.0 | 10x |
| 17.5 | |
| - 14.0 | 2x |
| 3.5 | |
| - 3.5 | 0.5x |
| 0 | |

Answer : 12.5

+ - + - + - + - + - + - +